EMLS

ELECTROMAGNETIC LEVITATION SYSTEM

User Manual

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1. INTRODUCTION

The electromagnetic levitation system controls the magnetic field generated by an electromagnet to levitate a small permanent magnet in midair. With an appropriate controller in the loop, the small magnet levitates in the air indefinitely without any disturbance. The vertical position of the levitating magnet is measured using a linear Hall effect sensor and the current in the electromagnet is actively controlled to achieve stable levitation.

The electromagnetic levitation system is one of the classical experimental setups to illustrate some of the analysis and design methods in control system education. It is also used to evaluate novel analysis and design methods in control system research. The system is highly nonlinear, open-loop unstable and extremely challenging to control robustly.

The electromagnetic levitation system has been developed to offer a low-cost experimental electromagnetic levitation setup for students, educators and researchers. The developed system is modeled very accurately and it is fully compatible with the HILINK (hardware-in-the-loop real-time control platform for Matlab/Simulink) and RAPCON (real-time rapid control prototyping platform for Matlab/Simulink) platforms.

1.1. System Features

- Fascinating demonstrations and experiments
- Hall effect position sensing
- Compatible with the HILINK and RAPCON platforms control platform for Matlab/Simulink)
- Accurate model with precise model parameters
- Compact and low-cost
- Works with both disc magnet and sphere magnet
- Magnet levitates in midair with an impressive air gap (approximately 2.5 cm)
- Highly efficient (no overheating)

1.2. Possible Experiments

- System modeling and identification
- Simulation and real-time systems
- PD, PID, lead controller design
- Root locus, time-domain and frequency-domain analysis and design

- Reference tracking and disturbance rejection
- State-space design (LQR, LQG, observer, $\mathcal{H}_2$, $\mathcal{H}_\infty$, μ-synthesis, sliding-mode, adaptive)
- Intelligent control (neural networks, fuzzy logic, genetic algorithms)
- Nonlinear system analysis and control

1.3. Requirements

- PC with Windows XP or later
- Matlab R2007b or later with Simulink, Real-Time Workshop and Real-Time Windows Target
- HILINK (or RAPCON) platform 1.0 or later
- Power supply ($6 - 15$ V, 2 A)

1.4. Specifications

- Electromagnet: 15.10 mH ferrite core coil
- Magnet: 41.3 g or 39.9 g barium-strontium magnet
- Sensor: 50 V/T Hall effect sensor
- Size: approximately 6.4 cm $\times$ 6.4 cm $\times$ 15.2 cm
- Weight: approximately 500 g

1.5. Connections

- Electromagnet: red $\rightarrow$ positive (A), blue $\rightarrow$ negative (B)
- Sensor: green $\rightarrow$ VDD, black $\rightarrow$ GND, white $\rightarrow$ OUT

2. SYSTEM MODEL

The electromagnetic levitation system is essentially made up of an electromagnet, a levitating magnet and a Hall effect sensor. Its model is shown in Figure 1, where R is the resistance of the coil, L is the inductance of the coil, v is the voltage across the electromagnet, i is the current through the electromagnet, m is the mass of the levitating magnet, g is the acceleration due to gravity, d is the vertical position of the levitating magnet measured from the bottom of the electromagnet, f is the force on the levitating magnet generated by the electromagnet and e is the voltage across the Hall effect sensor.

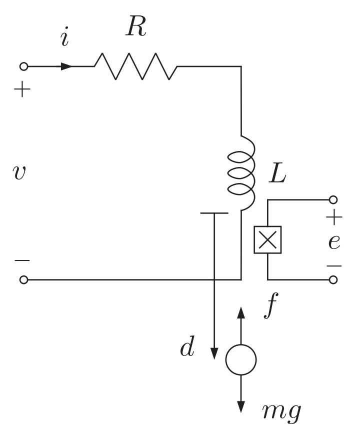

Figure 1. Electromagnetic levitation system model.

The force applied by the electromagnet on the levitating magnet can be closely approximated as

$$f = k\frac{i}{d^3}, \tag{2.1}$$

where k is a constant that depends on the geometry of the system [1]. The voltage across the Hall effect sensor induced by the levitating magnet and the electromagnet can be closely approximated as

$$e = \alpha + \beta\frac{1}{d^2} + \gamma i + n, \tag{2.2}$$

where α, β and γ are constants that depend on the Hall effect sensor used as well as the geometry of the system and n is the noise process that corrupts the measurement [2]. It follows from Newton's second law that

$$m\ddot{d} = mg - k\frac{i}{d^3}. \tag{2.3}$$

Moreover, it follows from the Kirchhoff's voltage law that

$$v = Ri + L\dot{i}. \tag{2.4}$$

Letting $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} d & \dot{d} & i \end{bmatrix}$ be the state of the system, $z = d$ be the controlled output, $y = e$ be the measured output, $u = v$ be the control input and $w = n$ be the disturbance/noise input, the standard state equation description of the system can be written as

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} x_2 \\ -\frac{k}{m}\frac{x_3}{x_1^3} + g \\ -\frac{R}{L}x_3 + \frac{1}{L}u \end{bmatrix}, \\ z &= x_1, \\ y &= \beta\frac{1}{x_1^2} + \gamma x_3 + \alpha + w. \end{aligned} \tag{2.5}$$

The equilibrium point of the system is at

$$\begin{bmatrix} x_{1e} \\ x_{2e} \\ x_{3e} \end{bmatrix} = \begin{bmatrix} (\frac{ku_e}{gmR})^{1/3} \\ 0 \\ \frac{u_e}{R} \end{bmatrix}, \tag{2.6}$$

where u_e is the required equilibrium electromagnet voltage to suspend the levitating magnet at $x_{1e} = d_e$. Note that there is a unique equilibrium point.

The Jacobian linearization of the system about the equilibrium point is

$$\begin{aligned} \delta\dot{x} &= A\delta x + B_1\delta w + B_2\delta u, \\ \delta z &= C_1\delta x + D_{11}\delta w + D_{12}\delta u, \\ \delta y &= C_2\delta x + D_{21}\delta w + D_{22}\delta u, \end{aligned} \tag{2.7}$$

where $\delta x = x - x_e$, $\delta w = w - w_e$, $\delta u = u - u_e$, $\delta z = z - z_e$, $\delta y = y - y_e$ and

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 & 0 \\ \frac{3g(gmR)^{1/3}}{(ku_e)^{1/3}} & 0 & -\frac{gR}{u_e} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} -\frac{2\beta gmR}{ku_e} & 0 & \gamma \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 1 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \end{bmatrix}. \end{aligned} \tag{2.8}$$

Note that $w_e = 0$, $z_e = x_{1e}$ and $y_e = \beta/{x_{1e}}^2 + \gamma x_{3e} + \alpha$.

The transfer matrix of the linearized system is

$$P(s) = \begin{bmatrix} 0 & \dfrac{-\dfrac{gR}{u_e L}}{(s+\dfrac{R}{L})[s^2 - \dfrac{3g(gmR)^{1/3}}{(ku_e)^{1/3}}]} \\ 1 & \dfrac{\dfrac{\gamma}{L}s^2 + [\dfrac{2\beta g^2 mR^2}{Lku_e^2} - \dfrac{3\gamma g(gmR)^{1/3}}{L(ku_e)^{1/3}}]}{(s+\dfrac{R}{L})[s^2 - \dfrac{3g(gmR)^{1/3}}{(ku_e)^{1/3}}]} \end{bmatrix}. \tag{2.9}$$

Note that

$$\begin{bmatrix} \Delta Z(s) \\ \Delta Y(s) \end{bmatrix} = P(s) \begin{bmatrix} \Delta W(s) \\ \Delta U(s) \end{bmatrix}, \tag{2.10}$$

where $\Delta Z(s)$, $\Delta Y(s)$, $\Delta W(s)$ and $\Delta U(s)$ are the Laplace transforms of $\delta z(t)$, $\delta y(t)$, $\delta w(t)$ and $\delta u(t)$, respectively.

In this derivation, the back emf induced by the moving levitating magnet is ignored as it is very small. If the Hall effect sensor is placed below the levitating magnet, then γ also becomes very small and it can also be neglected. Moreover, the force between the levitating magnet and the core of the electromagnet is ignored here for simplicity, and it should be taken into account for a more accurate model.

3. MODEL PARAMETERS

Letting the desired d_e be 2.00×10^{-2} m and using measurements obtained by a HILINK platform, the parameters of the electromagnetic levitation system are determined as

$$\begin{array}{lll}
R = 1.71\ \Omega, & L = 15.10 \times 10^{-3}\ \mathrm{H}, & m = 41.30 \times 10^{-3}\ \mathrm{kg}, \\
g = 9.81\ \mathrm{m/s^2}, & k = 3.10 \times 10^{-6}\ \mathrm{kg\,m^4/s^2/A}, & \alpha = 2.48\ \mathrm{V}, \\
\beta = 4.25 \times 10^{-4}\ \mathrm{V\,m^2}, & \gamma = 0.31\ \mathrm{V/A}, & u_e = 1.79\ \mathrm{V}, \\
x_{1e} = 2.00 \times 10^{-2}\ \mathrm{m}, & x_{2e} = 0.00\ \mathrm{m/s}, & x_{3e} = 1.05\ \mathrm{A}, \\
z_e = 2.00 \times 10^{-2}\ \mathrm{m}, & y_e = 3.87\ \mathrm{V}, & w_e = 0.00\ \mathrm{V}.
\end{array} \tag{3.1}$$

With these parameter values, it follows that

$$\begin{array}{l}
A = \begin{bmatrix} 0 & 1 & 0 \\ 1.4709 \times 10^3 & 0 & -9.3716 \\ 0 & 0 & -113.2450 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 66.2252 \end{bmatrix}, \\
C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \end{bmatrix}, \\
C_2 = \begin{bmatrix} -106.1254 & 0 & 0.3100 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 1 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \end{bmatrix},
\end{array} \tag{3.2}$$

and

$$P(s) = \begin{bmatrix} 0 & \dfrac{-620.6334}{(s + 113.2450)(s^2 - 1.4709 \times 10^3)} \\ 1 & \dfrac{20.5298 s^2 + 3.5667 \times 10^4}{(s + 113.2450)(s^2 - 1.4709 \times 10^3)} \end{bmatrix}. \tag{3.3}$$

The noise process n can be modeled as a white noise with spectral height

$$N = 1.00 \times 10^{-9}\ \mathrm{V^2\,s}. \tag{3.4}$$

4. EXPERIMENTS

Two possible experiments with the electromagnetic levitation system are outlined below. Throughout these experiments, it is assumed that the output of the Hall effect sensor is measured by one of the analog inputs of the HILINK board to determine the position of the levitating magnet and the electromagnet is driven by one of the H-bridges of the HILINK board. In addition, it is assumed that the supply voltage of the HILINK board is 12 V.

4.1. System Model

- Measure the resistance and inductance of the coil to determine R and L.
- Measure the mass of the magnet to determine m.
- When the magnet is far away from the system, apply $v = 0$ V to the electromagnet and measure the steady-state sensor output voltage e to determine α from $e = \alpha$.
- When the magnet is far away from the system, apply $v = 1$ V to the electromagnet and measure the steady-state sensor output voltage e to determine γ from $e = \alpha + \gamma v/R$.
- Suspend the magnet with an appropriate controller $d = 2$ cm away from the electromagnet, measure the steady-state sensor output voltage e to determine β from $e = \alpha + \beta/d^2 + \gamma v/R$, and use the voltage applied to the electromagnet v to determine k from $mg = kv/R/d^3$.

Questions

- What is a mathematical model and why is it important in engineering?
- Using the equations for the electromagnetic levitation system, describe a method to measure the parameters of the system.
- Propose a method to determine the resistance and inductance of the coil.
- Observe the waveform applied to the electromagnet by an oscilloscope and derive an equation between the voltage that the electromagnet "feels" and the applied voltage.

4.2. Controller Design

- Using the linearized model of the electromagnetic levitation system obtained, design a controller to suspend the magnet 2 cm away from the electromagnet.
- Connect the electromagnet and sensor to the HILINK board.
- Implement the controller in a Simulink model and run it.
- Show that the controller satisfies the design specifications.

- Change some of the parameters of the controller while the system is running and observe the corresponding change in the response.

Questions

- Describe a method for determining the step response of a system using the HILINK platform.
- Why is the step response important in control engineering?
- Determine the type of the system with the designed controller.
- Describe a method to measure the frequency response of the system.

REFERENCES

[1] D. K. Cheng, *Field and Wave Electromagnetics*. MA: Addison-Wesley, 1983.

[2] A. Smaili and F. Mrad, *Applied Mechatronics*. MA: Oxford, 2008.